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# Constraining the New Aether: gravitational Cherenkov radiation

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Abstract: We study the simplest concrete theory for spontaneous Lorentz violation, the "New Aether theory" of Jacobson and Mattingly, which is a vector-tensor gravitational theory with a fixed-modulus condition on the vector field. We show that the observation of ultra-high energy cosmic rays (which implies the absence of energy loss via various Cherenkov type processes) places constraints on the parameters of this theory, which are much stronger than those previously found in the literature and are also stronger than the constraints generically arising when gravity displays sub-luminal propagation.

Keywords: Space-Time Symmetries, High Energy Cosmic Rays.

## Contents



## 1. Introduction

Lorentz invariance is a cornerstone of modern particle physics and of general relativity. Exactly how well constrained is violation of Lorentz invariance? This question has received increasing experimental [1] and theoretical [2] scrutiny recently (for a nice review see [3]). Partly this is because one should always test the foundations of theories. Partly it is also because Lorentz violation is one possible route by which very high energy (Planck scale) physics might be detectable at the energy scales actually available experimentally; we can constrain (rule out or perhaps find evidence for) theories involving the highest energy scales if those theories happen to predict any deviation from Lorentz invariance. The study of Lorentz invariance violation has also been stimulated by the apparent non-observation of the GZK cutoff in the cosmic ray flux [4] (for a recent review see [5]).

Constraints on Lorentz violating physics are already quite severe. In fact, if Lorentz invariance is not a symmetry at the fundamental scale, then it is generically communicated to low energy scales in an unsuppressed manner [6, 7], leading to $O(10^{-2})$ violations in experiments which instead provide $< 10^{-22}$ bounds on its violation. Similarly, constraints on high dimension Lorentz violating operators show that they are excluded if they arise at the Planck scale, even with coefficients suppressed by several orders of magnitude [8, 9]. (Supersymmetry provides a potential way around this constraint [10].)

Therefore, the study of Lorentz violation has shifted towards investigating the spontaneous violation of Lorentz invariance, arising at a scale smaller than the Planck scale. If this violation occurs in a way which distinguishes between different species of matter, then the constraints again become very severe; so one line of work has concentrated on spontaneous Lorentz violation due to a field coupled only to gravitation.

Spontaneous Lorentz violation will occur whenever a field, transforming nontrivially under the Lorentz group, takes on a vacuum expectation value. This is the case, for instance, in the vector-tensor theories originally investigated by Will and Nordtvedt in the early 1970's [11]. Recently, a new twist on this idea, called the "New Aether theory," has been proposed ([12, 13], see also [14]). In this theory, besides the metric $g_{\mu\nu}$ and ordinary matter fields, there is a new vector field $S^\mu$, which is constrained, either by a potential term or by a Lagrange multiplier, to take on a fixed length. The most general lagrangian up to second order in derivatives for such a theory is,

$$S = \frac{1}{16\pi G_{\rm N}} \int d^4x \sqrt{-g} \left( -R - \mathcal{L}_V - \lambda (g_{\alpha\beta} S^\alpha S^\beta - v^2)^2 + \mathcal{L}_M \right), \tag{1.1}$$

with $\mathcal{L}_M$ the matter lagrangian (for instance, of the Standard Model) and

$$\mathcal{L}_V = \frac{c_1}{v^2} \nabla_\alpha S^\beta \nabla^\alpha S_\beta + \frac{c_2}{v^2} \nabla_\alpha S^\alpha \nabla_\beta S^\beta + \frac{c_3}{v^2} \nabla_\alpha S^\beta \nabla_\beta S^\alpha + \frac{c_4}{v^4} S^\alpha \nabla_\alpha S^\beta S^\mu \nabla_\mu S_\beta \,, \tag{1.2}$$

which is the most general kinetic term quadratic in derivatives. (We follow the notational conventions of [12, 13] for the $c_i$, which is opposite that of [15].) The potential term will ensure that the vector field $S$ will take a nonzero timelike vacuum expectation value, $S^2 = v^2$ (we use $[+,-,-,-]$ metric conventions), which then selects a preferred frame, the frame where the expectation value is purely in the time direction. In the limit $\lambda \to \infty$, this is a strict constraint; at finite but large $\lambda$, the "radial" fluctuation is heavy (though we will argue below that the finite $\lambda$ theory is not viable).

This "New Aether" theory is hardly the unique theory for studying the spontaneous violation of Lorentz symmetry; but it is the simplest, and its study may teach us about what physical phenomena best constrain such theories, and what physical problems are encountered in even trying to formulate them. Therefore we consider it an interesting problem to study what the tightest physical limits on this theory are. Previous studies have focused on cosmological tests [16], with bounds of order $10^{-1}$ on the $c_i$, and on the contribution of $S$ to the tightly constrained post-newtonian parameter $\alpha_2$ [15], which puts a constraint of order $10^{-7}$ on a complicated function of the $c_i$, constraining the $c_i$ to be $O(10^{-7})$ (except in special parameter ranges, where there can be cancellations). However, we expect that a much stronger constraint can be placed on this theory by calculating how efficiently relativistic particles will radiate gravitational and $S$-field excitations. One then uses the arrival of very high energy cosmic rays to place a bound on how efficient such energy loss can be, and therefore on the theory's parameters. Similar considerations provide the tightest constraints on subluminal propagation of gravity [17]. However, the application to this problem is richer because there are five propagating modes, corresponding roughly to the two transverse gravitational modes and two transverse and one longitudinal mode of the $S$ field [13], and any of these can be emitted by a high energy cosmic ray.

The purpose of this paper is to consider such processes and to use them to set a limit on the coefficients of the New Aether theory. As we shall see, the constraints are extremely strong, and represent the most stringent restrictions on this theory to date. Presumably this is because, wh¡ile previous studies of this theory [12, 13, 16, 15] have generally been based on considering it as a classical theory, our treatment takes the theory to be valid semi-classically (in particle physics language, at tree level). While Einsteinian gravity is not consistent as a fully quantum theory, it is consistent at this same semi-classical level, so we consider it reasonable to ask for the New Aether theory to be interpreted as a semi-classical, and not just classical, theory.

The outline of the remainder of the paper is as follows. In section 2, we make some remarks about the model, including bounds arising from causality, and the difficulties encountered above the scale where radial excitations occur. In section 3 we derive the constraint on the coefficients arising from the emission of traceless-tensor modes from high energy hadrons; section 4 obtains a similar constraint when the $S$ field modes are considered, and section 5 obtains another constraint from the emission of pairs of such excitations-a constraint which persists in the limit that all $c_i$ are taken to zero at fixed ratio. Finally, we present a summary of our findings in the discussion section.

## 2. Remarks on the New Aether theory

Before discussing constraints on the New Aether theory arising from a specific process, we will briefly mention some structural constraints on the theory.

To get a taste for the problems at hand, consider the theory in the absence of the potential term, $\lambda = 0$, and without a vacuum expectation value (using $c_i$ rather than $c_i/v^2$ as coefficients on the different terms). In this case, the back-reaction on the metric can be neglected. Since we are also temporarily not interested in the case with nonzero vacuum value, we can neglect the $c_4$ term in eq. (1.2). The resulting effective lagrangian is,

$$\mathcal{L}_{\rm IR} = -c_1 \partial_\mu S_\nu \partial^\mu S^\nu - (c_2 + c_3) \partial_\mu S_\nu \partial^\nu S^\mu \,, \tag{2.1}$$

where we replaced covariant with ordinary derivatives, since the metric is taken to be flat, and integrated by parts to make the $c_2$ term look like the $c_3$ term.

The point is that, even at the classical level, this lagrangian yields a theory with an energy which is not bounded from below, except in the special case $c_1 + c_2 + c_3 = 0$. Therefore, as soon as interactions of any sort are included, we expect runaway behavior. The fastest way to see this is to consider the case where $c_2 = c_3 = 0$ so only the first term is present; then express it in non-covariant language as

$$\mathcal{L} = -c_1 \Big( \partial_i S_j \partial_i S_j - \partial_0 S_j \partial_0 S_j - \partial_i S_0 \partial_i S_0 + \partial_0 S_0 \partial_0 S_0 \Big) \,. \tag{2.2}$$

This looks like an ordinary kinetic term for three fields named $S_1$, $S_2$, and $S_3$, and a wrong-sign kinetic term for a fourth field named $S_0$. The associated hamiltonian has positive energies associated with gradients and time derivatives of the $S_i$, but negative energies associated with $S_0$:

$$H = c_1 \Big( \partial_i S_j \partial_i S_j + \partial_0 S_j \partial_0 S_j - \partial_i S_0 \partial_i S_0 - \partial_0 S_0 \partial_0 S_0 \Big) \,. \tag{2.3}$$

This is the hamiltonian density derived by finding canonical momenta for each $S_\mu$ field and applying a Legendre transform, but it does not coincide with the hamiltonian density $T_{00}$

derived by variation with respect to $g^{00}$. However, the difference is a total derivative, the total energy remains unbounded from below, and the theory is unstable.

The contribution to $T_{00}$ arising from the $(c_2+c_3)$ term also leads to an unstable theory, but as is well known, the instabilities cancel and the hamiltonian is positive semidefinite for the special "gauge invariant" case $(c_1+c_2+c_3) = 0$. In fact, this is the reason that, when discussing vector fields in flat-space quantum field theory, gauge invariance is required in any renormalizable, interacting theory.

The theory with a vacuum expectation value but without a potential is also unstable except in the gauge invariant case. Classically, as soon as there are excitations in the gravitational and $S$ field degrees of freedom, the nonlinear interactions will allow a runaway in which positive energy excitations combined with the negative energy excitations associated with the wrong-sign component of $S$ grow without bound. In the quantum theory, even in vacuum and at tree level, there will be spontaneous production of a pair of positive and a pair of negative energy excitations; the rate is unbounded and this process can be used to rule out the theory, in analogy to "phantom" scalar theories [18].

Adding a potential term $-\lambda(S_\mu S^\mu - v^2)^2$ with finite $\lambda$ does not rescue the theory. Expanding $S^\mu = vu^\mu + \delta S$, the potential introduces a mass term for $\delta S^0$,

$$\mathcal{L} \supset -4\lambda v^2 (S_0)^2 \,, \tag{2.4}$$

but since the kinetic term for $S_0$ has the wrong sign, this actually makes the field tachyonic. For instance, if $c_2 = 0 = c_3$, the full lagrangian for $S_0$ is

$$(-4\lambda v^2)S_0^2 + \partial_i S_0 \partial_i S_0 - \partial_0 S_0 \partial_0 S_0 \quad \Rightarrow \quad \omega^2 = -4\lambda v^2 + k^2 \,. \tag{2.5}$$

Changing the sign of $\lambda$ does not help because it destabilizes the $S_i$ fields, which then have a negative quartic term.

The only way out is to replace the potential with a strict constraint, $S_\mu S^\mu = v^2$. This removes one degree of freedom (perturbatively, $S_0$) from the model, leaving a positive definite hamiltonian. For this reason, in this paper we will take the New Aether theory to have the condition $S_\mu S^\mu = v^2$ imposed as a strict constraint. Of course, this nonlinear sigma model is not renormalizable, which means it lacks a good interpretation on scales larger than $v$ beyond a semi-classical (tree) level. However, the same is true of gravity and the scale $m_{\rm pl}$, so we will not take this as a fundamental objection to considering this theory (though the lower $v$ must be, the more of an embarrassment it becomes that the theory cannot be considered at the loop level above this scale).

Jacobson and Mattingly have derived the normal modes of propagation of the theory, their speeds of propagation, and their admixture between graviton and $S$ field in a particular gauge [13]. Their results are reproduced, with a few approximations, in table 1. For future reference, the complete expressions for the transverse and trace propagation speeds are,

$$s^2_{\rm transverse} = \frac{2c_1 - c_1^2 + c_3^2}{2(c_1+c_4)(1-c_1-c_3)} \,, \tag{2.6}$$

and

$$s^2_{\rm trace} = \frac{(c_1+c_2+c_3)(2-c_1-c_4)}{(c_1+c_4)[2(1+c_2)^2 - (c_1+c_2+c_3)(1+c_1+2c_2+c_3)]} \,, \tag{2.7}$$

| Wave mode | squared speed $s^2$ | polarization (when $\hat{k} = \hat{z}$) |
|---|---|---|
| transverse, traceless metric | $1/(1-c_1-c_3)$ | $h_{12}$, $h_{11} = -h_{22}$ |
| transverse Aether | $\simeq \frac{c_1}{c_1+c_4}$, see eq. (2.6) | $h_{i3} \simeq [(c_1+c_3)/s]S_i$, $i = 1, 2$ |
| trace | $\simeq \frac{c_1+c_2+c_3}{c_1+c_4}$, see eq. (2.7) | $h_{00} = -2\,\delta S_0$, $h_{33} \simeq 2\,\delta S_0/s^2$ |

**Table 1:** Wave mode speeds and polarizations in the gauge $h_{0i} = v_{i,i} = 0$, from [13].

while the complete expressions for $h_{ii}$ in the trace mode are

$$h_{11} = h_{22} = -(c_1 + c_4)\,\delta S_0\,, \qquad h_{33} = \frac{(1 + c_2)(c_1 + c_4)}{c_1 + c_2 + c_3}\,2\delta S_0\,. \tag{2.8}$$

We end this section by mentioning what restrictions can be placed on the coefficients of this theory by demanding stability, the absence of ghosts, and causality. This has been addressed previously (see for instance [19, 15]), and we have fairly little to add, except to note how easily the constraints follow from these speeds of propagation of the normal modes. As shown in the table, there are three distinct propagating modes with different dispersion relations. In the limit where the $c_i$ are small (the relevant limit, since large values are excluded), these can be understood as the standard transverse traceless graviton mode, the transverse $S$ field mode $\delta\vec{S}\cdot\vec{k} = 0$, and the longitudinal $S$ field mode, $\delta\vec{S} \parallel \vec{k}$ (in the frame where the $S$ field condensate is in the time direction, which we will use throughout).

Gradient instabilities appear when a squared speed of propagation is negative; from the table, this imposes the constraints,

$$\frac{c_1}{c_1+c_4} \geq 0\,, \qquad \frac{c_1+c_2+c_3}{c_1+c_4} \geq 0\,. \tag{2.9}$$

Further, the kinetic $(\partial_0 S_i)^2$ term is wrong sign and the $S$ field excitations are "ghostlike" (and ruled out by the arguments of [18]) unless

$$c_1+c_4 \geq 0\,, \tag{2.10}$$

and therefore $c_1 \geq 0$ and $(c_1+c_2+c_3) \geq 0$ separately.

We impose in addition the constraint that propagation speeds not be super-luminal. After all, we are considering a metric theory, which should be causal in the usual sense. This constraint imposes,

$$\frac{c_1}{c_1+c_4} \leq 1\,, \qquad \frac{c_1+c_2+c_3}{c_1+c_4} \leq 1\,, \qquad c_1+c_3 \leq 0\,. \tag{2.11}$$

More precisely, the first two constraints should be derived from eq. (2.6) and eq. (2.7). Lim has argued [19] that these constraints are essential, since their violation could allow propagation around closed curves. However they cannot be considered to be on as firm ground as the other constraints.



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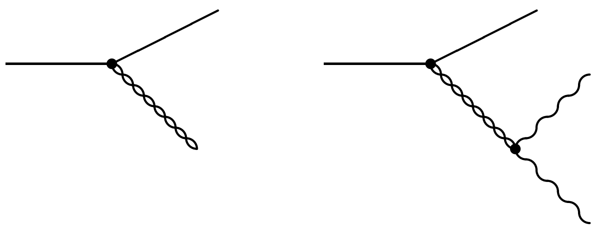

**Figure 1:** Left: gravitational Cherenkov radiation. Right: emission of a pair of $S$ particles via a virtual graviton.

The constraints we find will be on the sizes of the $c_i$. $S_\mu$ can be re-scaled to force $v^2 = 1$, which has been done in the previous literature. In this case, one would say that we are constraining the kinetic terms of $S$ to be small. Alternately, we can rescale $S_\mu$ to make the $c_i/v^2$ be order 1, say, $c_1/v^2 = 1/2$, so the kinetic term for transverse excitations is canonically normalized; then the vacuum expectation value $v^2 = 2c_1$ is small, and we would say we are finding constraints on the size of the vacuum expectation value of the field $S$. We prefer this language, since it makes it more clear that what is being constrained is the size of the spontaneous Lorentz violation. However, previous authors have generally chosen $v^2 = 1$, and we will continue to define the $c_i$ to follow their notation.

## 3. Constraints from Gravi-Cherenkov radiation

All constraints we impose on the New Aether theory will arise by considering the possibility that high energy cosmic rays should undergo a spontaneous emission process producing one or more excitations of the coupled gravity-$S$ field modes. That is, we shall consider diagrams of the form shown in figure 1, in which the solid line is either a fermion or a gauge boson, with a dispersion relation we shall take to be lightlike, and the double-wiggly line is a graviton. First we consider the former diagram, with a graviton in the final state.

Normally this process is impossible, because there is no way to conserve both the energy and the momentum of the final state particles and still have them on-shell. This would be true if they both satisfied massive or light-like dispersion relations. However, we are investigating a theory where Lorentz symmetry is broken, and the dispersion relations are different. Since in particular the transverse traceless graviton modes will now have sub-luminal dispersion, with $k_3^2 = k_0^2/s^2$, $s^2 \leq 1$ the propagation speed in the preferred frame, these modes carry more momentum than energy, and it is kinematically allowed to emit one. The larger a particle's energy in the preferred frame (assumed to be the frame defined by the microwave background radiation[1]), the higher the emission rate; so

[1]If the preferred frame differs from the frame of the microwave background, our bounds actually become stronger. This is because high energy cosmic rays have been observed arriving from many directions. In any frame at a large boost $\gamma \gg 1$ with respect to the microwave frame, some of these high energy cosmic rays were more energetic than in our frame; our results should be applied to these cosmic rays.



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the arrival of extremely high energy cosmic rays sets a limit on how far from light-like the gravity dispersion relation can be.

The problem of calculating the rate for this process has previously been solved by Moore and Nelson [17], but we will repeat the derivation here giving somewhat more detail, as well as making a more careful treatment of the partonic structure of the cosmic ray hadrons which turns out to slightly strengthen the constraint derived.

Consider the emission of gravitons by a fermionic parton inside of a hadron. (Since the energy scale involved in the problem is large, a partonic discussion is necessary, see for instance [9]). In the gauge used by Jacobson and Mattingly [13], the emitted excitation is purely a graviton. The vertex between fermion and graviton is (see for instance [20])

$$\frac{-i\sqrt{16\pi G_{\rm N}}}{8}\left[\gamma_\mu(P_{1\nu}+P_{2\nu})+\gamma_\nu(P_{1\mu}+P_{2\mu})-2\eta_{\mu\nu}(\not{P}_1+\not{P}_2)\right], \tag{3.1}$$

where $P_1$ and $P_2$ are the 4-momenta of the two external fermion states. (Our notation will be that capital letters are 4-momenta, while lower case letters are the magnitudes of their spatial parts.) Since the matter lagrangian is unchanged by including the $S$ field, this vertex is the same; what is different is the dispersion relation of the final state graviton, which has maximum speed $s = 1/\sqrt{1-c_1-c_3} \equiv 1-\epsilon$, with $\epsilon \simeq (-c_1-c_3)/2 \ll 1$. (The assumption $\epsilon \ll 1$ is self-consistent, since we are deriving a constraint on $\epsilon$ which will show that it is order $10^{-15}$.)

Labeling the incoming momentum $P$ and the outgoing graviton and fermion momenta $K$ and $P-K$, we then have $P^2 = 0 = (P-K)^2$ and $K^2 = -2\epsilon k^2$, where $k = |\vec{k}|$ is the magnitude of the graviton spatial momentum in the preferred frame. It follows that $P \cdot K = -\epsilon k^2$.

The total emission rate of gravitons from this parton is

$$\Gamma = \frac{1}{2p}\int \frac{d^4K}{(2\pi)^4}\, 2\pi\,\delta[(P-K)^2]\, 2\pi\,\delta[s^{-2}k_0^2-|\vec{k}|^2] \sum |\mathcal{M}^2|\,, \tag{3.2}$$

where $\sum|\mathcal{M}|^2$ is the squared matrix element, summed over final and averaged over initial spins and polarizations, and we have used the energy-momentum conserving delta function to perform the integral over the fermionic final state momentum. Since the dispersion relation in the on-shell delta function for the graviton is only simple in the preferred frame, this is the best frame to evaluate the integral. The delta functions can be used to reduce the $K$ integration to two integrals, over $k$ and $\phi$ a trivial azimuthal angle;

$$\frac{1}{2p}\int \frac{d^4K}{(2\pi)^4}\, 2\pi\,\delta[(P-K)^2]\, 2\pi\,\delta[(1+2\epsilon)k_0^2-k^2] = \frac{1}{32\pi^2p^2}\int_0^p dk \int_0^{2\pi} d\phi\,, \tag{3.3}$$

with $K^2 = -2\epsilon k^2$ and $pk - \vec{p}\cdot\vec{k} = \epsilon k(p-k)$.

The matrix element is easiest to evaluate in the frame where $\vec{k}$ defines the $z$ axis; in this frame, if we choose $\vec{p}$ to lie in the $x-z$ plane, $p_x^2 = 2\epsilon p(p-k)$. We find it most convenient to compute separately the contribution of the $h_{11}-h_{22}$ and $h_{12}+h_{21}$ polarizations, with the former giving

$$\sum|\mathcal{M}|^2 = \frac{16\pi G_{\rm N}}{128}(4p_x)^2\,{\rm Tr}(\not{P}-\not{K})\gamma^x\not{P}\gamma^x = 16\pi G_{\rm N}\epsilon^2 p(p-k)(4p^2-4pk+k^2)\,, \tag{3.4}$$

and the latter giving

$$\sum |\mathcal{M}|^2 = \frac{16\pi G_{\rm N}}{128}(4p_x)^2\, {\rm Tr}(\not{P}-\not{K})\gamma^y \not{P}\gamma^y = 16\pi G_{\rm N}\epsilon^2 p(p-k)k^2\,. \tag{3.5}$$

Summing these, and then doing the trivial $\phi$ integral, the integrated rate is,

$$\frac{32\pi G_{\rm N}\epsilon^2}{16\pi p^2}\int_0^p dkp(p{-}k)(2p^2-2pk+k^2) = \frac{3}{2}G_{\rm N}\epsilon^2 p^3\,. \tag{3.6}$$

Similarly, a gauge boson interacts via the vertex [20]

$$\frac{-i\sqrt{16\pi G_{\rm N}}}{2}\left(P_1\cdot P_2 C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(P_1,P_2) + \text{Gauge dependent}\right), \tag{3.7}$$

where the gauge dependent part vanishes when contracted on external polarizations, $\mu\nu$ are the graviton polarization indices, $\rho$ is the Lorentz index for the boson with momentum $P_1$, $\sigma$ is the Lorentz index for the boson with momentum $P_2$, and

$$C_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\,, \tag{3.8}$$

$$D_{\mu\nu,\rho\sigma}(P_1,P_2) = \eta_{\mu\nu}P_{1\sigma}P_{2\rho} - \left[\eta_{\mu\rho}P_{1\nu}P_{2\rho} + \eta_{\mu\rho}P_{1\sigma}P_{2\nu} - \eta_{\rho\sigma}P_{1\mu}P_{2\nu} + (\mu\leftrightarrow\nu)\right]. \tag{3.9}$$

The kinematics for graviton emission from a gauge boson are the same as for a fermion, but the matrix element differs; we find the matrix element squared, summed over physical final polarizations and averaged over physical initial polarizations, is

$$\sum |\mathcal{M}^2| = 16\pi G_{\rm N}\epsilon^2\left[(2p(p{-}k){+}k^2)^2 + k^2(2p{-}k)^2\right], \tag{3.10}$$

leading to an emission rate of

$$\Gamma = \frac{12}{5}G_{\rm N}\epsilon^2 p^3\,. \tag{3.11}$$

The process discussed would cause ultra-high energy cosmic rays to lose energy. If the energy loss timescale is shorter than the propagation time across the galaxy $\sim 10$ Kpc (the shortest scale on which we believe cosmic rays must propagate-arguably the highest energy cosmic rays propagate tens of megaparsecs), then the cosmic rays would not reach us; so the time scale for the highest energy cosmic rays to propagate without substantial energy loss must be longer than 10 Kpc $\sim 10^{36}\,{\rm GeV}^{-1}$. The transverse momentum "kick" a parton experiences in emitting a graviton, found above, is of order $Q^2 \sim p^2\epsilon$; this is also the scale on which parton distribution functions should be evaluated. We will see in a moment that this is about $Q^2 \sim (3000\,{\rm GeV})^2$; such a transverse momentum kick is enough to cause a hadron to fragment, so a single graviton emission should be taken to destroy the hadron, and the relevant rate is the total rate of graviton emission. This must be determined by summing over the partons in the hadron. We saw that the emission rate scales as $p^3$, so the relevant rate is

$$\Gamma = G_{\rm N}\epsilon^2 p^3 \sum_f \int_0^1 dx x^3 f(x)\times \begin{cases} 3/2 & \text{quark} \\ 12/5 & \text{gluon}\,. \end{cases} \tag{3.12}$$

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The right value of $Q^2$ to evaluate the parton distribution functions $f(x)$ is determined self-consistently from the value of $\epsilon$ saturating the bound; taking the hadron responsible for the highest energy cosmic ray to be a proton, and using CTEQ parton distribution functions [21], we find $\sum \int x^3 f(x) \times (3/2 \,\text{or}\, 12/5) \simeq 0.025$. The result is dominated by the fermion, even though $(3/2) < (12/5)$, because the $x^3$ weighted PDF is about 6 times larger for a quark. With this in mind we will neglect gluon contributions in the following sections.

Considering that the highest energy cosmic ray which has been detected arriving at the earth had an energy of $p \sim 3 \times 10^{11}$ GeV [22], and requiring that it was able to propagate 10 Kpc, we find a bound,

$$\epsilon \equiv \frac{-c_1 - c_3}{2} < 5 \times 10^{-16} \,. \tag{3.13}$$

This bound is somewhat stronger than the one quoted in [17], because we took a graviton emission to fragment the hadron, whereas their treatment just summed up the energy losses due to such emissions. Note that the emission rate depends on the departure of the dispersion relation from luminal, $\epsilon$, as $\epsilon^2$. This should be compared to ordinary Cherenkov radiation, where the energy loss rate goes as $\epsilon$. This is a spin effect. The polarization vector/tensor is transverse and only sensitive to the transverse component of the momentum, which is $O(\epsilon^{1/2})$. For a spin-1 particle this appears at first order in the amplitude, for a spin-2 particle it appears at second order. This will be important in what follows, because the $S$-field polarization states are lower spin, and will therefore be less suppressed in the $\epsilon \to 0$ limit.

## 4. Emission of $S$ modes

The New Aether theory has five physical propagating modes, rather than the two familiar from pure general relativity. This can be simply understood; three degrees of freedom (the four $S_\mu$ modes, minus the mode removed by the fixed length condition) have been added. At small $c_i$, a gauge can be found in which the propagating modes divide into the two transverse traceless graviton modes and three $S$ field modes.

The approximate propagation speeds of these three modes are easily read off from the lagrangian, eq. (1.2), by replacing covariant with ordinary derivatives, and by setting $S^\mu \nabla_\mu \to \partial_0$ in the last term;

$$\mathcal{L}_V = S_i \left( \frac{-c_1}{v^2}(k_0^2 - k_l k_l \delta_{ij}) - \frac{-c_2 - c_3}{v^2} k_i k_j + \frac{-c_4}{v^4} k_0^2 \right) S_j \,. \tag{4.1}$$

Two degenerate modes correspond to $S_i k_i = 0$, the transverse Aether modes, with dispersion relation $(c_1/v^2 + c_4/v^4)k_0^2 = (c_1/v^2)k^2$. The remaining mode has $S_i \parallel k_i$; we shall call it a longitudinal mode. Its dispersion relation is $(c_1/v^2 + c_4/v^4)k_0^2 = ([c_1{+}c_2{+}c_3]/v^2)k^2$. These reproduce the small $c$ limits presented in table 1.

If we consider the $v$-scaled $c_i$ to be $O(1)$ coefficients and $v^2 \ll m_{\rm pl}^2$, then it is natural to treat $c_4 \sim c_1^2$. This interpretation is natural if the fundamental scale is the Planck scale but the vacuum expectation value is smaller. For this reason we will take the transverse mode to propagate close to the speed of light. Under this approximation, we need the full expression, eq. (2.6), for its speed of propagation $s^2$:

$$s^2_{\rm transverse} = \frac{2c_1 - c_1^2 + c_3^2}{2(c_1{+}c_4)(1 - c_1 - c_3)} \simeq 1 + \frac{c_1^2 + 2c_1c_3 + c_3^2 - 2c_4}{2c_1} \equiv 1 - \epsilon' \,. \tag{4.2}$$

Here $\epsilon' \sim c_i$ is a small parameter indicating the failure of the transverse mode to move at the speed of light.

When gravity is included, each mode takes on a small admixture of graviton, and therefore couples to standard model particles and can be emitted as radiation in processes analogous to the one considered in the last section. To compute the emission rate, we must include the leading order $h_{\mu\nu}$ content of the excitation to find the matrix element, and we must correctly account for the particle's spectral weight (correctly normalize the external state) in writing down the external phase space.

We begin with the transverse $S$ excitation. The correct on-shell delta function accounting for the particle's spectral weight (correctly normalizing the external state) is

$$2\pi\, \delta\Big(2(c_1{+}c_4)k_0^2 - 2c_1|\vec{k}|^2\Big) \,, \tag{4.3}$$

which introduces a $1/c_1$ factor in the rate. The vertex with a fermion is eq. (3.1) contracted with $h_{13} = h_{31} = (c_1{+}c_3)/s$ (see table 1) for one polarization and $h_{23} = h_{32} = (c_1{+}c_3)/s$ for the other. Therefore $|\mathcal{M}|^2$ introduces two positive factors of $(c_1{+}c_3)$ and the result is suppressed by $c$ relative to the result for a graviton. However, as we will see, the matrix element will contribute only one, not two, factors of $1 - s^2$, so the rate is just as large as for a spin-2 graviton.

Using the vertex and polarization just presented, choosing coordinates such that only $k_3$ is nonzero, the matrix element is,

$$\mathcal{M} = \frac{i\sqrt{16\pi G_{\rm N}}(c_1{+}c_3)}{4} \bar{u}(P{-}K)\,[\gamma_1(2P - K)_3 + \gamma_3(2P - K)_1]\,u(P) \tag{4.4}$$

for one polarization and $1 \to 2$ for the other polarization. The spin summed, squared matrix element is

$$\sum |\mathcal{M}|^2 = 4\pi\epsilon' G_{\rm N}(c_1{+}c_3)^2\left(32p^4 - 32p^3k + 42p^2k^2 - 10pk^3 + k^4\right), \tag{4.5}$$

where $\epsilon'$ is the correction from lightlike dispersion for the transverse mode, defined above. The total production rate from a fermion is obtained by integrating over $k$ and is

$$\Gamma = \frac{277}{80}\frac{(c_1{+}c_3)^2(c_1^2 + 2c_3c_1 + c_3^2 - 2c_4)}{2c_1^2} G_{\rm N}p^3 \,. \tag{4.6}$$

We have not computed the analogous expression for gluons because the parton distribution function of gluons leads to a smaller contribution.

Using the same integral over parton distributions as in the last section and applying this bound to the highest energy cosmic ray observed, we get a constraint,

$$\frac{(c_1{+}c_3)^2(c_1^2 + 2c_1c_3 + c_3^2 - 2c_4)}{2c_1^2} < 7\times 10^{-32} \,. \tag{4.7}$$

Since the quantity constrained is $\sim c^2$, this constraint is of comparable strength to the previous one-but has an independent functional dependence on the $c_i$.

Now we turn to the longitudinal $S$ mode. The dispersion relation for this mode, $s^2 \simeq (c_1{+}c_2{+}c_3)/(c_1{+}c_4)$, is the only one which generically deviates by an $O(1)$ amount from $s^2 = 1$ even in the limit of small $c_i$ (small vacuum value $v$ for the $S$ field). It will therefore lead to a strong constraint. But as we will see, the constraint is even strong when $s^2 \simeq 1$, because there is no spin suppression in the emission of this particle in this limit.

We begin with the phase space. In the small $c_i$ limit and in a gauge where the gauge condition is based only on $h_{\mu\nu}$, the energy-momentum conserving delta function correctly accounting for the spectral weight (right external state normalization) of the longitudinal mode is

$$2\pi\delta\Big(2(c_1{+}c_4)k_0^2 - 2(c_1{+}c_2{+}c_3)|\vec{k}|^2\Big)\,, \tag{4.8}$$

leading to a phase space integration of

$$\begin{aligned}
\frac{1}{2p}\int \frac{d^4P_f d^3k dk_0}{(2\pi)^8} 2\pi\delta(P_f^2) 2\pi\delta\Big[2(c_1{+}c_2{+}c_3)k^2 - 2(c_1{+}c_4)k_0^2\Big](2\pi)^4\delta^4(P{-}P_f{-}K) = \\
= \frac{1}{2p}\int \frac{k^2 dk\, d\cos\theta_{pk}\, d\phi}{(2\pi)^3}\frac{1}{4|c_1{+}c_2{+}c_3|k}2\pi\delta\Big[(p{-}sk)^2{-}p^2{-}k^2{-}2pk\cos\theta_{pk}\Big] \\
= \frac{1}{32\pi|c_1{+}c_2{+}c_3|p^2}\int_0^{\frac{2}{1+s}p} dk\,,
\end{aligned} \tag{4.9}$$

where $s^2 = (c_1{+}c_2{+}c_3)/(c_1{+}c_4)$ is the propagation speed.

However, Jacobson and Mattingly's results [13] for the mixing of the graviton and $S$ field in the state polarization are in a gauge where $\partial_i S_i = 0$. To use the polarization vectors found there, we must relate the amplitude of $\delta S_0$ in this gauge to the amplitude of $\delta S_{\hat{k}}$ in the above gauge. The relation turns out to be

$$\delta S_0(\text{Jacobson's gauge}) = s\delta S_{\hat{k}}(\text{above gauge})\,. \tag{4.10}$$

Therefore the matrix element squared will contain an extra factor of $s^2$ if the matrix element is computed normalizing the vertex based on $S_0$, which means $|c_1{+}c_2{+}c_3|$ should be replaced with $|c_1{+}c_4|$.

The results we have written for the dispersion relation and polarization tensor in table 1 are insufficient to compute the matrix element because it vanishes at this order;

$$\bar{u}(P-K)\left[\gamma_0(2P-K)_0 - \frac{1}{s^2}\gamma_3(2P-K)_3\right]u(P) \propto \bar{u}(P-K)\left[\gamma_\mu K^\mu\right]u(P) = 0\,. \tag{4.11}$$

Therefore we must use the full expressions for the dispersion relations and the polarization tensor, eq. (2.7) and eq. (2.8), which expanded to the desired order become (choosing $\hat{k}$ as the $z$ axis)

$$h_{00} = -s^2 h_{33} - (c_2{+}c_4{-}c_3)S_0\,, \qquad h_{11} = h_{22} = -(c_1{+}c_4)S_0\,. \tag{4.12}$$

These lead to a matrix element squared of

$$\sum|\mathcal{M}|^2 = 4\pi G_{\rm N}(2p-ks)^2(4p^2 - 4spk - k^2(1-s^2))(c_3{-}c_4)^2\,, \tag{4.13}$$

and a total rate of

$$\Gamma = \frac{2}{15} G_{\mathrm{N}} p^3 \frac{(c_3-c_4)^2}{|c_1+c_4|} \frac{6s^3+24s^2+35s+20}{(1+s)^4}\,, \tag{4.14}$$

with $s=\sqrt{(c_1+c_2+c_3)/(c_1+c_4)}$. If we had not expanded in small $c_i$, then $(c_3-c_4)^2$ above would instead read $(c_3(2+c_4)-2c_4+c_1(c_1+c_3+c_4))^2/4(1-c_1-c_3)^2$.

The function of $s$ above ranges from 20 to 5.3125 as $s$ goes from 0 to 1; for simplicity we will set it to 5, which will give a conservative bound. Since $p_\perp^2$ can be as large as $p^2$ for this process if $s$ is small, we also use the parton distribution functions evaluated at $10^{10}$ GeV, which give $\int x^3 f(x) = .0052$ for quarks. Using the same constraint on $\Gamma$ as before, we find

$$\frac{(c_3-c_4)^2}{|c_1+c_4|} < 1\times 10^{-30}\,. \tag{4.15}$$

Curiously, the emission rate which leads to this constraint does *not* vanish in the limit $s\to 1$ that the emitted mode approaches lightlike propagation. This is because its emission, unlike transverse $S$ and graviton emission, is not spin suppressed. It can also be understood as an example of non-decoupling of a longitudinal mode of a vector field when the mass becomes zero but gauge invariance is broken. However, when $(1-s)<10^{-22}$, then it becomes important that the proton dispersion is not precisely lightlike; so the above limit is conditional on $(c_2+c_3-c_4)/c_1 > 10^{-22}$.

The emission rate also has a nonzero limit as $s$ goes to zero, $(c_1+c_2+c_3)\to 0$, which is the "gauge invariant" limit for the kinetic term under which the theory remains free of tachyons if the strict fixed-modulus limit on the $S$ field is lifted. However, if the strict fixed modulus limit is lifted and $\lambda v^2 \lesssim (10^{10}\,\mathrm{GeV})^2$, then the treatment of the external propagating states must be reconsidered. Note that, as $s\to 0$, the energy lost by emitting a longitudinal $S$ mode goes to zero. Naively, this means that the emission process does not degrade the energy of the emitter, and is therefore allowed. The emission process can still be ruled out, however, because it causes the breakup of the hadron, so no single final state particle carries as much energy as the initial hadron, and the highest energy cosmic rays would still be degraded in energy.[2]

## 5. $\psi \to \psi SS$

The other process we will consider is the emission of *two* $S$ field excitations via an off-shell graviton propagator, the diagram on the right in figure 1. This process is suppressed, with respect to the processes we have just discussed, by an additional power of $G_{\mathrm{N}}$, so naively it would not be important. However, the processes we considered all vanish as the $c_i$ are taken to zero; the graviton dispersion approaches the light-cone, preventing its emission, and the direct coupling of matter to $S$ field excitations (via their mixing with

[2] If the highest energy cosmic rays are elementary particles such as photons, rather than hadrons, all of our bounds are much stronger, since $\int dx x^3 f(x)$ is approximately 1 in this case. For the $s\to 0$ limit, energy loss occurs whenever the $S$ emission is accompanied by bremsstrahlung or pair production. The inelastic cross-section for photon primaries is large enough that the energy loss rate still exceeds that for hadrons.

the graviton) is suppressed and also vanishes linearly in $c$. In contrast, the coupling of the graviton to matter and to (canonically normalized) $S$ fields remains finite as the $c_i$ are taken to zero. In this limit, the transverse $S$ field excitation becomes lightlike and the phase space for its production disappears; but the longitudinal $S$ excitation remains sub-luminal with $s^2 = (c_1{+}c_2{+}c_3)/(c_1)$ and its production persists. Therefore it is possible to set a limit on (ratios of) the $c_i$ which persists even as the $c_i$ are taken to *zero* (with ratios held fixed).

In studying this process, we will systematically expand in small $c_i$, which is justified by the results of the previous sections. We will also take $c_4 \ll c_1$, which we previously argued is reasonable. Were we not to make this approximation, then transverse $S$ modes could also be pair produced, and we would find that this process implies that $c_4/c_1$ is indeed small. Since the $c_i$ are small, we can systematically neglect the mixing of graviton and Aether excitations.

We begin by making a crude parametric estimate of the production rate. At generic values of $s^2$, the squared speed of the longitudinal mode, the production rate should be of order

$$\Gamma \sim \frac{G_{\rm N}^2 p^5}{16\pi}\,, \tag{5.1}$$

purely on parametric grounds. Substituting $p \sim 10^{11}$ GeV, this gives $\Gamma \sim 10^{-23}$ GeV. This is to be compared with the propagation time of a typical cosmic ray, $\sim 10^4$ Kpc $\sim 10^{36}$/GeV. The mean lifetime of a cosmic ray before emitting an $S$ pair is much shorter than the propagation time. Therefore all parameters are excluded which do not make $1 - s^2 \ll 1$, and we are justified to expand in this quantity.

### 5.1 Kinematics

We begin with a treatment of the kinematics of the reaction. The theory has a preferred frame, and it is awkward not to work in this frame. Since this is not the rest frame of the emitting particle, and since the usual Lorentz invariance tricks for treating phase space cannot be used, the treatment of the kinematics is somewhat more complicated than would usually be the case for an integrated 3-body decay rate. We choose the coordinate axes such that the momentum of the initial particle is along the z-axis. The four-momenta of the 3 final-state particles are written as $\mathbf{k}_1$ and $\mathbf{k}_2$ for the two longitudinal $S$ modes and $\mathbf{k}_3$ for the mode with lightlike dispersion. All but 5 final state variables are performed by energy-momentum conserving delta functions. We choose to leave as integration variables the following 5 independent variables:

$$k_{1z}\,, \quad k_{3z}\,, \quad |\mathbf{k}_{3\perp}|\,, \quad \beta\,, \quad \text{and} \quad \phi\,,$$

where $\beta$ is the angle between $\mathbf{k}_{1\perp}$ and $-\mathbf{k}_{3\perp}$. The angle $\phi$ corresponds to an overall rotation around the z-axis and cannot be fixed by the conservation of momentum. At the same time, this angle does not appear anywhere in the matrix elements involved in the emission processes, so the integral over this independent variable is trivial.

We need expressions for the other variables in the problem in terms of these 5 independent variables. However, we do not need absolutely general expressions, because we can make two approximations. First, defining

$$s \equiv 1 - \epsilon \,, \tag{5.2}$$

we have $\epsilon \ll 1$. (The estimate, eq. (5.1), shows that $\epsilon \sim 1$ is already excluded, so we may take $\epsilon \ll 1$ in trying to determine the limit on $\epsilon$.) This means that, at generic $k_{1z}$ and $k_{3z}$, the $\mathbf{k}$ are all collinear, and we may expand to quadratic order in the $\mathbf{k}_\perp$. Second, the graviton propagator will have a virtuality of $(P - K_3)^2$. When $\mathbf{k}_{3\perp}$ is small, this is close to zero, and the matrix element is enhanced by this nearly on-shell intermediate state. For this reason, the dominant region of the integration is at small $\mathbf{k}_{3\perp}$, and we may expand in this quantity being smaller than $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$.

Under the collinear approximation, energy conservation reads

$$k_{1\perp}^2\left(\frac{1}{k_{1z}} + \frac{1}{P{-}k_{1z}{-}k_{3z}}\right) + k_{3\perp}^2\left(\frac{1}{k_{3z}} + \frac{1}{P{-}k_{1z}{-}k_{3z}}\right) - 2\frac{k_{1\perp}k_{3\perp}\cos\beta}{P{-}k_{1z}{-}k_{3z}} - 2\epsilon(p - k_{3z}) = 0\,. \tag{5.3}$$

Further approximating that $\mathbf{k_3}_\perp \ll \mathbf{k_1}_\perp$, we find,

$$k_{1\perp}^2 = k_{z1}k_{z2}\sqrt{2\epsilon} = k_{2\perp}^2\,. \tag{5.4}$$

To make the notation in the remainder of the section more compact, we will define $x = \frac{p-k_{3z}}{p}$, $y = \frac{k_{1z}}{p}$, $z = \frac{k_{3\perp}}{p}$. $x$, $y$, and $z$ are independent integration variables, with $x$ in the range $[0,1]$, $y$ in the range $[0,x]$, and $z$ ranging up to of order $\sqrt{\epsilon}$, but as discussed, the rate will be dominated by the region where $z \ll \sqrt{\epsilon}$, so the upper bound of this integration will not be important. The $\beta$ integral runs over the range $[0, 2\pi]$ for small $z$, and at leading order in small $z$ the integrand will not depend on $\beta$, so this integral may also be conducted trivially.

With these simplifications, the phase space becomes

$$\begin{aligned}
&\frac{1}{2}\frac{1}{2p}\int\frac{d^3k_1\,d^3k_2\,d^3k_3}{(2\pi)^9 8k_1k_2k_3}(2\pi)^4\delta(k_{1z}+k_{2z}+k_{3z})\delta(p{-}k_1^0{-}k_2^0{-}k_3^0)\delta^2(\mathbf{k}_{1\perp}{+}\mathbf{k}_{2\perp}{+}\mathbf{k}_{3\perp}) \simeq \\
&\quad\simeq \frac{1}{2^{10}\pi^5 p}\int\frac{dk_1dk_3}{k_1k_2k_3}\,d\phi\,d\beta\,k_{1\perp}dk_{1\perp}\,k_{3\perp}dk_{3\perp}\,\delta\left(\frac{k_{1\perp}^2}{2}\left[\frac{1}{k_1}+\frac{1}{k_2}\right] - 2\epsilon(p-k_3)\right) \\
&\quad= \frac{p}{2^8\pi^3}\int_0^1 dx\int_0^x dy\int dz\frac{z}{x(1-x)}\,,
\end{aligned} \tag{5.5}$$

where the small $k_{3\perp}$ approximation was made in passing from the first to second expressions. The leading factor of $1/2$ in the first expression is the final state symmetry factor.

### 5.2 Matrix element

The vertex coupling the graviton to two $S$ field excitations can be found by expanding eq. (1.2) to linear order in $h_{\mu\nu}$. As discussed above, we neglect $c_4$. Writing the vertex as $h_{\mu\nu}(Q)S_\alpha(K_1)S_\beta(K_2)V^{\mu\nu\alpha\beta}$, we find

$$\frac{V^{\mu\nu\alpha\beta}}{\sqrt{16\pi G_{\rm N}}} = -c_1\left[\eta^{\alpha(\mu}\eta^{\nu)\beta}\left(2K_1\cdot K_2 + Q^2\right) + 2\eta^{\alpha\beta}K_1^{(\mu}K_2^{\nu)} + 2(K_1{-}K_2)^{(\mu}\eta^{\nu)[\alpha}\,Q^{\beta]}\right] -$$

$$
\begin{aligned}
&-c_2\Big[2K_1^\alpha(K_2+Q)^{(\mu}\eta^{\nu)\beta} - K_1^\alpha Q^\beta\eta^{\mu\nu} + 2K_2^\beta(K_1+Q)^{(\mu}\eta^{\nu)\alpha} - K_2^\beta Q^\alpha\eta^{\mu\nu}\Big] - \\
&-c_3\Big[2K_1^\beta K_2^{(\mu}\eta^{\nu)\alpha} + 2K_2^\alpha K_1^{(\mu}\eta^{\nu)\beta} + Q^2\eta^{\alpha(\mu}\eta^{\nu)\beta} + 2(K_1-K_2)^{(\mu}\eta^{\nu)[\alpha}\,Q^{\beta]}\Big]\,.
\end{aligned}
\tag{5.6}
$$

This is linear in the $c_i$ and so the matrix element squared is quadratic in the $c_i$; however, there is a $1/(2c_1)$ factor associated with each $S$ field external state, see eq. (4.3), so the rate is zero order in the $c_i$. Alternatively, if we canonically normalize the $S$ field from the start, each term in eq. (5.6) should be multiplied by $1/(2c_1)$.

In calculating the matrix element we must be somewhat careful, since the final state phase space remains non-vanishing even as the graviton becomes lightlike. Since the matrix element will be singular at this point due to the graviton propagator, one must treat the kinematics of the very small $k_{3\perp}$ region carefully, in particular including the nonzero proton mass in computing $Q^2$, how far off-shell the graviton is. (Because of this near-singularity, the dominant $|Q^2|$ will be $\lesssim m_{\rm pr}^2$ the proton mass squared. The process is therefore not probing the structure of the proton, and we can consider it as a single fermionic object.) This gives corrections of the form $m_{\rm pr}^2/p^2$ to the virtuality of the graviton.

We again use the expression for the graviton-matter coupling from [20] and take the graviton propagator in the usual Dedonder gauge. The longitudinal polarization vectors are adequately approximated by

$$
\vec{\epsilon}_1 \simeq \vec{\epsilon}_2 \simeq \hat{z}\,, \tag{5.7}
$$

good at leading order in $\epsilon^{1/2}$. Therefore,

$$
\begin{aligned}
\vec{\epsilon}_1\cdot\vec{p} &= \vec{\epsilon}_2\cdot\vec{p} \simeq p\,, \\
\vec{\epsilon}_1\cdot\vec{k}_3 &= \vec{\epsilon}_2\cdot\vec{k}_3 \simeq p(1-x)\,.
\end{aligned}
\tag{5.8}
$$

Because of the near-singularity in the graviton propagator, the cross-section is dominated by the kinematic regime $Q^2 \sim m_{\rm pr}^2 \ll \epsilon p_0^2 \sim K_1\cdot K_2 \sim P\cdot K_1$. In this regime most of the terms in eq. (5.6) are subdominant; only the terms involving $c_1 K_1\cdot K_2$, $c_2(K_1^\alpha K_2^\mu + K_2^\beta K_1^\mu)$, and $c_3(K_1^\alpha K_2^\mu + K_2^\beta K_1^\mu)$ contribute at order $\epsilon p_0^2$. Furthermore, since we are specializing to the regime $\epsilon \ll 1$, we must have $|c_2{+}c_3| \ll c_1$; otherwise the propagation speed is far from luminal, which we have already excluded. Therefore the $c_2$ and $c_3$ contributions to the matrix element cancel in the leading kinematic regime and we are left with only the $c_1$ contribution. This rather substantially simplifies the calculation. The matrix element, including the final state normalization factor mentioned above, reads

$$
\sum|\mathcal{M}|^2 = \frac{(16\pi G_N)^2}{4Q^2Q^2}\,(K_1\cdot K_2)^2\,(\hat{z}\cdot\vec{p})(\hat{z}\cdot\vec{k}_3)\left(\hat{z}\cdot\vec{p}+\hat{z}\cdot\vec{k}_3\right)^2. \tag{5.9}
$$

In terms of the variables defined in the preceding section this is

$$
\sum|\mathcal{M}|^2 = \varepsilon^2(16\pi G_{\rm N})^2 p^4\,\frac{(1{-}x)^3\,(2{-}x)^2\,[y^2+(x{-}y)^2]^2}{4\left(z^2+\frac{m_{\rm pr}^2}{p^2}x^2\right)^2}\,, \tag{5.10}
$$

which is to be substituted into eq. (5.5). The $z$ integral gives,

$$
\int\frac{z\,dz}{\left(z^2+\frac{m_{\rm pr}^2}{p^2}x^2\right)^2} = \frac{p^2}{2m_{\rm pr}^2x^2}\,, \tag{5.11}
$$

illustrating that the integral is dominated by small $z$ as claimed. The remaining $x$ and $y$ integrations are straightforward, and give a total rate of

$$\Gamma = \frac{\epsilon^2 p^7 G_{\rm N}^2}{225\pi m_{\rm pr}^2} \,. \tag{5.12}$$

Unlike the processes in the previous two sections, in this process the transverse momentum "kick" to the proton is relatively small, so the proton typically does not break up, and we should compute the rate of energy loss rather than the frequency of emission. This is done by adding a factor $xp$ inside the integral, leading to an energy loss per time of

$$\frac{dp}{dt} = \frac{29\epsilon^2 p^8 G_{\rm N}^2}{14400\pi m_{\rm pr}^2} \,. \tag{5.13}$$

Assuming that the energy loss should not exceed the total energy of the highest energy cosmic ray observed over a 10 Kpc distance, and again using $p = 3 \times 10^{11}$ GeV [22], this leads to a bound on $\epsilon$ of

$$\epsilon \equiv \frac{c_4 - c_2 - c_3}{c_1} < 3 \times 10^{-19} \,. \tag{5.14}$$

We see that the approximation $(m_{\rm pr}^2/p^2) \ll \epsilon$ is self-consistent.

## 6. Discussion and conclusions

Spontaneous Lorentz violation requires that a field, transforming nontrivially under the Lorentz group, take on a vacuum expectation value. The simplest available field is a vector field, and the most general theory where such a field takes on a vacuum value, which is stable and free of ghosts, is the New Aether theory, with a lagrangian given in eq. (1.1), eq. (1.2). This theory is ill behaved unless the restriction on the modulus $S_\mu S^\mu$ is imposed as a constraint, not through a potential. This renders the theory non-renormalizable and valid only semi-classically.

The most notable feature of the theory at the semi-classical level is that there are 5 propagating modes — two graviton, two transverse $S$, and one longitudinal $S$—which are massless but have sub-luminal propagation. As a result, high energy particles moving close to the speed of light can radiate them, in analogy with the Cherenkov process. This makes high energy particles lose their energy — the higher the energy, the faster the energy loss. Since very high energy cosmic rays are known to travel astronomical distances, this places constraints on the theory. Namely, it cannot have parameters causing energy loss which would degrade those high energy cosmic rays. Since there are several modes with different dispersion, there are several constraints. In particular, we have found,

$$-c_1 - c_3 < 1 \times 10^{-15} \qquad \text{eq. (3.13)} \tag{6.1}$$

$$\frac{(c_1{+}c_3)^2(c_1^2 + 2c_1c_3 + c_3^2 - 2c_4)}{c_1^2} < 1.4 \times 10^{-31} \qquad \text{eq. (4.7)} \tag{6.2}$$

$$\frac{(c_3{-}c_4)^2}{|c_1{+}c_4|} < 1 \times 10^{-30} \qquad \text{eq. (4.15)} \tag{6.3}$$

$$\frac{c_4 - c_2 - c_3}{c_1} < 3 \times 10^{-19} \qquad \text{eq. (5.14)}\,. \tag{6.4}$$

The next-to-last bound is conditional on $(c_4 - c_2 - c_3)/c_1 > 10^{-22}$. In every case the opposite side of the expression is bounded by 0. This is because no dispersion relation can be super-luminal in flat space.

Combining these bounds with eq. (2.9)–(2.11), the $c_i$ are constrained such that each must be $|c_i| < 10^{-15}$, except for a special case in which each dispersion relation in table 1 is almost exactly luminal. Using the full expressions, eq. (2.6) and eq. (2.7) for the propagation speeds, this special case occurs when the $c_i$ satisfy

$$c_3 = -c_1\,, \qquad c_4 = 0\,, \qquad c_2 = \frac{c_1}{1 - 2c_1}\,. \tag{6.5}$$

We have checked that these same conditions cause $\alpha_2$ as determined in [15] to vanish (assuming the result quoted there is leading order in the $c_i$).

Physically, this means that Cherenkov processes rule out the New Aether theory unless either

1. parameters are just right such that all three propagating modes (graviton and two $S$ field) are light-like to extremely high precision, a 1-dimensional subspace of the 4-dimensional parameter space of the model, or

2. canonically normalizing the field $S$, the vacuum expectation value $v$ of $S$ satisfies $v < 3 \times 10^{-8} m_{\rm pl} \sim 5 \times 10^{11}$ GeV, or

3. the constraints, eq. (2.11), are not correct; there are modes which propagate super-luminally, despite the theory being a generally covariant metric theory of gravity. (We earlier discounted this possibility because we are considering a metric theory where the violation of Lorentz invariance is supposed to be spontaneous. However, the necessity of the constraint against super-luminal propagation is controversial, so we acknowledge this logical possibility.)

The constraints are not fatal to the theory, but they mean that either the theory does not predict Lorentz violating propagation speeds, or the scale of the spontaneous Lorentz violation must be much smaller than the Planck scale.

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